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Probabilistic Fatigue Crack Growth Analyses for Critical Structural Details

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The damage tolerance method requires the assumption that cracks preexist at the critical structural details and that subsequent fatigue crack propagation could cover at least twice the design life before failure occurs. If fatigue crack propagation cannot meet these requirements, an inspection could be arranged to detect cracks at half of the fatigue life to prevent catastrophic failure. Fatigue tests using from coupons up to the whole structure should be performed to complement the damage tolerance analyses. Nevertheless, the damage tolerance method can still not guarantee an economical airframe due to uncertainties in initial fatigue quality, material, usage, and maintenance. Probabilistic analyses are needed to cover major sources of these uncertainties. A procedure is presented based on several novel probabilistic fracture mechanics solutions for the analyses of fatigue crack propagation. A practical example is used in the discussion to illustrate the procedure and to address important issues when uncertainties are considered. It is shown that different consequences may appear if analyses of the fatigue life are based on the probabilistic consideration.

Introduction

TO guarantee enough structural strength during the design life span, the damage tolerance is usually considered for critical structural details to achieve the maximum reliability with a minimum maintenance requirement. The damage tolerance method requires specifying that cracks should be assumed in all of the primary structural details, and that these cracks should not grow to a size to cause loss of the structure within a specified service period. The damage tolerance method requires 1) a full-scale fatigue test to at least double the design life for an average usage load and 2) at least twice the design life for a crack started from a specified size to grow into the critical size. The damage tolerance method is often implemented by an inspection to detect fatigue cracks at half of the crack growth life.

The fracture-mechanics-based damage tolerance method provides information about the average crack growth behavior. It usually does not account for uncertainties involved in the fatigue crack growth process for practical structural problems. Empirical safety factors are often adopted to cover uncertainties in crack propagation. When there are no adequate data to support selection of a safety factor, a large value is preferred, which may lead to uneconomical heavy structures and to rejection of reliable structures and components. The consequent maintenance programs are usually poorly defined, resulting in uneconomical inspection intervals that may still not guarantee a reliable structure in service. A probabilistic analysis of the fatigue crack growth is desirable in the reliability management of structural problems. The analysis can be used as a guideline for service conditions, where various uncertainties cannot be avoided.

When accidental damages such as pilot error, maintenance damage, environmental attacks, production faults, etc., are excluded, probabilistic fatigue crack growth analyses should cover the effect of predictable variables such as initial flaws, stochastic crack growth, crack growth threshold, production and loading, failure criteria, inspection, fleet size, etc. In this paper, a crack growth analytical procedure is developed based on Elber's fatigue crack closure model[1] for nonstationary stochastic fatigue crack growth analyses under the general loading condition. The experimental data for the fatigue crack growth rate against the effective stress intensity factor for constant amplitude loading are used as intrinsic material data to represent the resistance of material to the fatigue crack growth. The stress, which is a function of both the applied load and geometry, is considered as a crack growth driving force. Together with a strip yield crack closure solution,[2] uncertainties in initial flaws, material, spectrum severity, geometry etc., are to be accounted for as a result of competition between the crack growth driving force and the material resistance in a simplified solution. This model makes it possible for the probabilistic analyses of fatigue crack growth in general structures under service conditions.

Crack Closure Analyses

In many of the stochastic fatigue crack growth solutions, the governing functions proposed for both constant and random loading are various randomized versions of the Paris–Erdogan[3] or compatible equations, which do not account for all of the parameters that affect crack growth even under the constant amplitude loading conditions. Considerable error may be introduced when such models are used to solve the crack propagation under spectrum loading, when load interaction, small crack growth, stress state effect, etc., should be considered. Other models, like those based on the reaction rate theory,[4] though attractive as microphysical-based models, still require much refinement to be used for the prediction of crack growth under variable amplitude loading.

Recent investigations based on the crack closure mechanism provide a new way to analyze the probabilistic fatigue crack growth. The crack closure mechanism is a phenomenon where a natural fatigue crack may remain closed during part of the loading even for tensile–tensile types of fatigue loading.[5] It has been revealed that various spectrum and stress state related crack growth behaviors are closely related to the crack closure mechanism.[6]

In accordance with the crack closure model, the crack growth rate may be approximated in the form of a piecewise-linear exponent function, as shown in Fig. 1:

$$w(K_{\max})\frac{\mathrm{d}a}{\mathrm{d}N} = C_i(\Delta K_{\mathrm{eff}})^{b_i} \tag{1}$$

where C_i and b_i are considered as material constants for each log linear regime, as shown in Fig. 1a. ΔK_{eff} is an effective stress intensity factor range computed from the crack opening load to the peak load and $w(K_{\max})$ is used to account for the quasi-static crack growth acceleration when the maximum stress intensity factor approaches the critical value (fracture toughness or plastic collapse). A general form can be written for the quasi-static acceleration function:

$$w(K_{\max}) = \left[1 - (K_{\max}/K_{\mathrm{cr}})^{\alpha}\right]^{\beta} \tag{2}$$

where K_{cr} is the critical stress intensity factor, which may be different from the static critical stress intensity factors such as the plane

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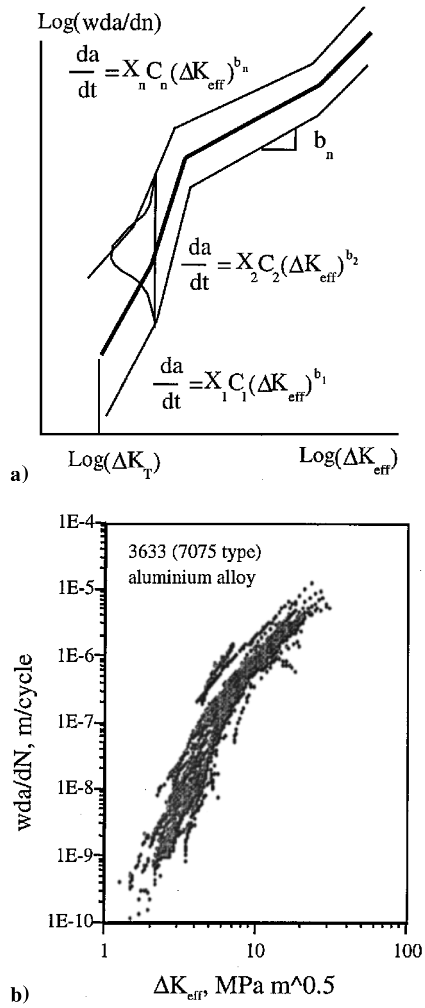


Fig. 1 Intrinsic crack growth rate relation and an example.

strain fracture toughness K_{Ic}, the plane stress fracture toughness K_c, or the fracture toughness K_e for partially through cracks.

The effective stress intensity factor range ΔK_{eff} is closely related to the reverse plastic yielding at the crack tip[7] that quantitatively represents the irreversible dislocation movement at the crack tip for the homogenous materials. The crack closure is thus far the most relevant parameter in characterizing the fatigue crack growth under variable amplitude fatigue loading conditions.

The effective stress intensity factor (Elber's[1] ΔK_{eff}) is determined by

$$\Delta K_{\text{eff}} = \sigma_{\text{eff}} Y(a)\sqrt{\pi a} \tag{3}$$

where a is the crack size, $Y(a)$ is a geometry function, and σ_{eff} is an effective stress range that depends on the stress range and the crack closure. Here, σ_{eff} is defined by

$$\sigma_{\text{eff}} = \begin{cases} (\sigma_{\max} - \sigma_{\text{op}}), & \text{if} \quad \sigma_{\text{op}} > \sigma_{\min} \\ (\sigma_{\max} - \sigma_{\min}), & \text{otherwise} \end{cases} \tag{4}$$

where σ_{op} is the stress level at which the crack tip begins to open.

Figure 1b shows an example of experimental crack growth rate expressed as a function of the effective stress intensity factor range for various stress ratios. The test data are gathered from different sources. There is significant scatter around the mean value. The scatter is due to both the material inhomogeneity and the test methods. Among the experimental uncertainties, the alignment of specimens may not be accurate, and the measurement of the crack size may be inaccurate. These random uncertainties will not seriously affect the mean crack growth rate when the sample size is large.

The difficulty in using the crack closure model arises for the evaluation of crack opening stress, especially under the spectrum loading condition. The crack opening stress may be different for every load cycle. By extending the Dugdale–Barenblatt's (see Ref. 8) strip yield model, it is now possible to make cycle-by-cycle fatigue crack growth analyses in a reasonable computation time[2,9] according to the crack closure model for practical problems.

Probability of Crack Initiation (POCI)

Fatigue is usually divided into the crack initiation and the propagation stage. The crack initiation is further divided into the crack nucleation and the small crack growth stage. The crack nucleation is often referred to as the stage when stress risers are created on a smooth surface due to fatigue loading. This stage is closely related to the early S-N (stress-cycle) fatigue tests for which smooth round small specimens are used extensively. Except for some dedicated applications for high surface quality and particular materials, the crack nucleation stage is often not significant because various flaws may be present on the initial surface due to impurities in materials, machine scratches, and environmental attacks. It is often adequate to consider the crack initiation stage to consist of only the small crack growth stage.

For many metallic materials, a fatigue crack growth threshold has been observed for certain stress levels below which the fatigue crack may stop growing. A threshold stress intensity factor range is often used to determine whether or not a crack may start to grow. The threshold intensity factor range often depends on the stress ratio and the environment under the constant amplitude loading condition. The value is difficult to determine when spectrum loading is considered. Recent development[10] indicates that an intrinsic threshold may be defined for a given material if the crack closure is considered. This intrinsic stress intensity factor is less dependent on stress ratio and load magnitude so that it can be used to deal with spectrum loading problems.

Based on the intrinsic fatigue crack growth threshold, a state function can be written to define whether or not an initial flaw may lead to a subsequent crack initiation and propagation:

$$g_T = \Delta K_T - \Delta S_{\text{eff}} Y(a_{\text{th}})\sqrt{\pi a_{\text{th}}} \tag{5}$$

where ΔK_T is the intrinsic threshold value, ΔS_{eff} is an effective stress range, Y is a geometry function, and a_{th} is the crack size at which the effective stress intensity factor reaches the minimum value. All of these parameters are random variables.

The probability of crack initiation (POCI) can be determined using the state function of Eq. (5) for the probability of all $g_T < 0$. Here, ΔS_{eff} represents approximately the external load variation. Y is a geometry variable, and a_{th} is the threshold crack size that may be larger than the physical initial flaw size because a crack may be arrested due to the buildup of crack closure.

In accordance with level II[11] analysis that requires only the solution for the central moments up to the second order, a reliability index β_T is computed for POCI as

$$\beta_T = \mu_{g_T} / \sigma_{g_T} \tag{6}$$

The reliability index is a ratio relating the mean value and the standard deviation of g_T. If random variable g_T can be assumed to follow a normal distribution, β_T can be used to evaluate POCI according to

$$\text{POCI} = \Phi(-\beta_T) \tag{7}$$

The mean value of g_T can be computed according to

$$\mu_{g_T} = E\{\Delta K_T - \Delta S_{\text{eff}} Y(a_{\text{th}})\sqrt{\pi a_{\text{th}}}\} = \Delta \bar{K}_T - (\Delta K_{\text{eff}})_{\min} \tag{8}$$

where $\Delta \bar{K}_T$ is the mean intrinsic threshold. The second term is the effective stress intensity factor range that can be determined by

$$(\Delta K_{\text{eff}})_{\min} = E\{\Delta S_{\text{eff}} Y(a_{\text{th}})\sqrt{\pi a_{\text{th}}}\} \tag{9}$$

where $(\Delta K_{\text{eff}})_{\min}$ is the minimum effective stress intensity factor in the crack propagation.

Suppose random variables ΔK_T, ΔS_{eff}, Y, and a_{th} are independent, the standard deviation of g_T can be calculated according to

$$(\sigma_{g_T})^2 = \text{var}\{\Delta K_T - \Delta S_{\text{eff}} Y \sqrt{\pi a_{\text{th}}}\} = \text{var}\{\Delta K_T\} + \text{var}\{\Delta S_{\text{eff}} Y \sqrt{\pi a_{\text{th}}}\} \tag{10}$$

where

$$\text{var}\{\Delta S_{\text{eff}} Y \sqrt{\pi a_{\text{th}}}\} = \text{var}\{(\Delta S_{\text{eff}} Y) \sqrt{\pi a_{\text{th}}}\} = \pi E^2\{\Delta S_{\text{eff}} Y\}\text{var}\{\sqrt{a_{\text{th}}}\} + \pi E^2\{\sqrt{a_{\text{th}}}\}\text{var}\{\Delta S_{\text{eff}} Y\} + \pi\, \text{var}\{\sqrt{a_{\text{th}}}\}\text{var}\{\Delta S_{\text{eff}} Y\} \tag{11}$$

for which $\text{var}\{\Delta S_{\text{eff}} Y\}$ and $\text{var}\{\sqrt{a_{\text{th}}}\}$ should be solved separately. Because ΔS_{eff} and Y are assumed to be independent, $\text{var}\{\Delta S_{\text{eff}} Y\}$ can be solved as

$$\text{var}\{\Delta S_{\text{eff}} Y\} = E^2\{\Delta S_{\text{eff}}\}\text{var}\{Y\} + E^2\{Y\}\text{var}\{\Delta S_{\text{eff}}\} + \text{var}\{Y\}\text{var}\{\Delta S_{\text{eff}}\} = (\Delta \bar{S}_{\text{eff}})^2\sigma_Y^2 + \bar{Y}^2\sigma_S^2 + \sigma_Y^2\sigma_S^2 = (\Delta \bar{S}_{\text{eff}})^2\bar{Y}^2(v_Y^2 + v_S^2 + v_Y^2 v_S^2) \tag{12}$$

where v_S and v_Y are coefficients of variation (COVs) for ΔS_{eff} and Y. By assuming that $\text{var}\{\sqrt{a_{\text{th}}}\}$ is approximated by a parabolic[12] function, then

$$\text{var}\{\sqrt{a_{\text{th}}}\} \approx (1/4\bar{a}_{\text{th}})\sigma_{a_{\text{th}}}^2 = 0.25\bar{a}_{\text{th}} v_{a_{\text{th}}}^2 \tag{13}$$

where $v_{a_{\text{th}}}$ is the COV of a_{th}. Therefore,

$$\text{var}\{\Delta S_{\text{eff}} Y \sqrt{\pi a_{\text{th}}}\} \approx \left[0.25 v_{a_{\text{th}}}^2 + v_Y^2 + v_S^2 + v_Y^2 v_S^2 + 0.25 v_{a_{\text{th}}}^2 (v_Y^2 + v_S^2 + v_Y^2 v_S^2)\right](\Delta K_{\text{eff}})_{\min}^2 = v_K^2 (\Delta K_{\text{eff}})_{\min}^2 \tag{14}$$

where

$$v_K^2 = 0.25 v_{a_{\text{th}}}^2 + v_Y^2 + v_S^2 + v_Y^2 v_S^2 + 0.25 v_{a_{\text{th}}}^2 (v_Y^2 + v_S^2 + v_Y^2 v_S^2) \tag{15}$$

The reliability index of POCI can be determined by

$$\beta_T = \frac{1 - (\Delta K_{\text{eff}})_{\min}/\Delta \bar{K}_T}{\sqrt{v_T^2 + v_K^2[(\Delta K_{\text{eff}})_{\min}/\Delta \bar{K}_T]^2}} \tag{16}$$

Among the COV, $v_{a_{\text{th}}}$ is a difficult variable to determine. It can be practically assumed to have the same value as the initial flaw distribution.

In the crack initiation analyses, all of the initial flaws are assumed to be able to propagate with different probability. A deterministic crack closure analysis can be performed for the crack starting at the mean initial flaw. Together with the random values of threshold, geometry, stress, and crack size, the probability of crack initiation can be estimated.

Nonstationary Stochastic Crack Growth

Even though extensive investigations have been performed for the statistical fatigue crack propagation under laboratory conditions,[13,14] the available models are usually valid for limited problems such as constant amplitude loading or some spectra with constant loading features. In practical applications, a total life prediction method is desired to evaluate the crack growth from the initial flaw to final failure. The analysis should, therefore, include the effect of the small crack growth behavior as well. Scatter in crack growth rates will increase for the small crack growth. The scatter will decrease when cracks become large. Apparently, the stationary stochastic process assumption cannot be used for a total life analysis. Nonstationary stochastic analysis is necessary. In this study, a new method is developed, instead of the Markov[15] or the differential nonstationary solutions. The method is based on the basic probabilistic consideration of damage accumulation for each load cycle. The new solution is an extension of the Palmgren–Miner[16] theory of linear cumulative damage.

The Palmgren–Miner's cumulative damage solution is a consequence of the theoretical statistical investigation for random loading by Miles[17] using an assumed S-N curve. When the Palmgren–Miner theory is used in the probabilistic analyses, it has been recognized[18] to be overconservative for spectrum loading histories. The model often fails to account for the crack growth under spectrum loading because it includes the whole load range as a primary damage parameter. This model does not distinguish the part of load cycle that effectively drives the crack growth and the part of load cycle that has no effect on crack growth. Large portions of the load cycle that have no effect on either crack growth or scatter are treated so as to have the same effect on the crack growth as the portions of the loads that do have effect on crack growth. Instead, by using the crack closure as a primary damage parameter, this S-N curve-based solution may be extended into a fatigue crack growth solution.

To approximate the nonstationary process, several stationary stochastic crack growth regimes may be assumed according to the fatigue crack growth rate as shown in Fig. 1. In each stationary regime, the number of load cycles can be determined according to the stationary stochastic solutions such as

$$n_i(a, n \mid a_i, N_i) = \int_{a_i}^{a_{i+1}} \frac{w(K_{\max})\,\mathrm{d}a}{g_i(\Delta K_{\text{eff}})} = \int_{N_i}^{N_{i+1}} X_i(N)\,\mathrm{d}N \tag{17}$$

where $X_i(N)$ is the stationary process for each regime (see Fig. 1a). The simple central moment solution as proposed by Yang and Manning[19] can be used together with the variable autocorrelation model proposed by Tanaka and Tsurui[20] to analyze the crack propagation in each stationary regime.

For the crack growth in the Paris regime (see Ref. 3), conventional stationary stochastic solutions can be used to account for the scatter in crack propagation caused by either the interspecimen variation or the stochastic crack propagation. When the crack grows across more than one stationary regime, a random variable solution can be derived so that the total fatigue cycles are determined by a sum of

$$n = n_1 + n_2 + \cdots + n_m \tag{18}$$

with a mean value of

$$E\{n\} = \sum_{i=1}^{m} E\{n_i\} \tag{19}$$

and a variation of

$$\text{var}\{n\} = \sum_{i=1}^{m}\sum_{j=1}^{m} \text{cov}\{n_i, n_j\} \tag{20}$$

A correlation can be defined among load cycles in various stationary regimes:

$$r_{ij} = \frac{\text{cov}(n_i, n_j)}{\sigma_{n_i}\sigma_{n_j}}, \quad \text{for} \quad r_{ii} = 1 \tag{21}$$

where r_{ij} is within a range of $|r_{ij}| \leq 1$. Here σ_{n_i} is the standard deviation in each stationary region that is determined by Eq. (17). This solution is required for the crack growth analysis across several stationary regimes especially near the threshold value, where variation in the crack growth rate may change significantly. A correlation function of r_{ij} may be assumed to be

$$r_{ij} = \exp(-|i - j|/\theta) \tag{22}$$

with parameter θ as a measurement of the correlation in the crack growth accumulation among different stationary regimes. This assumption features a close relation for the regimes close to each other, and a weak correlation for the regimes separated from each other. The parameter θ can be computed according to both long and short

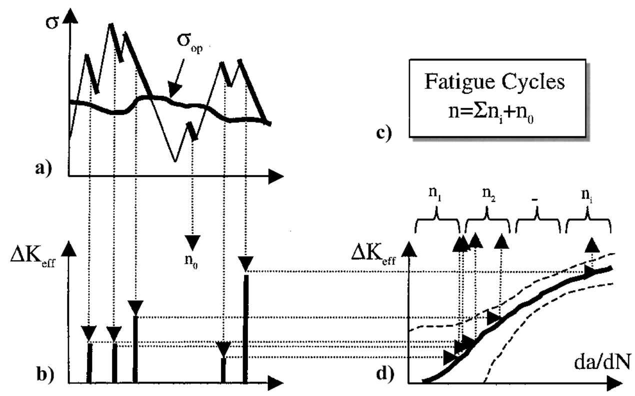


Fig. 2 Schematic of the damage accumulation solution based on the crack closure mechanism.

crack growth experimental data. The variation of total damage can, thus, be computed in the relation

$$\operatorname{var}\{n\} = \sum_{i=1}^{m} \sum_{j=1}^{m} r_{ij} \sigma_{n_i} \sigma_{n_j} \tag{23}$$

In this solution, the central moments can be determined for the fatigue cycles under spectrum loading condition. The schematic of the damage accumulation model is shown in Fig. 2. In the analysis, the crack closure is determined according to the mean crack size and the load history. The effective load range for each cycle is computed. The effective load range is then used together with the crack size to determine the effective stress intensity factor. The mean crack growth rate is estimated for each load cycle according to the intrinsic crack growth rate. The effective stress intensity factor range also determines the stochastic crack growth regime for each load cycle. The mean number of cycles in each regime can be determined, and the probabilistic analysis of the scatter in the final fatigue cycles can be realized according to Eqs. (17–23).

Effect of Load and Geometry

Thus far, the solution is to account for the effect of statistical quantities in the resistance of material under a deterministic crack growth driving force. This can only be achieved under laboratory conditions. We must further investigate how variations in applications, typically the variations in load severity and production, affect the fatigue crack growth. This analysis becomes relatively easy when the fatigue crack process is considered to be a competition between the crack growth driving force and the material resistance according to the crack closure mechanism. Now we concentrate on how variations in the applied loads or local stresses affect the driving force in the crack growth governing function.

In accordance with the crack closure mechanism, the crack growth driving force, the effective stress intensity factor ΔK_{eff}, is determined by the crack size, the crack opening stress, and the stress at the crack tip. In accordance with Eqs. (3) and (4), ΔK_{eff} can be written as

$$\Delta K_{\text{eff}} = \sigma_{\max} Y(a) \sqrt{\pi a} \begin{cases} (1 - \sigma_{\text{op}}/\sigma_{\max}), & \text{if} \quad \sigma_{\text{op}} > \sigma_{\min} \\ (1 - \sigma_{\min}/\sigma_{\max}), & \text{otherwise} \end{cases} \tag{24}$$

When the maximum stress $\sigma_{\max}$, the crack opening stress σ_{op}, and the geometry function $Y(a)$ are treated as random variables, representing variations in applied load, crack closure, geometry, aspect ratio of the crack, etc., the crack growth driving force ΔK_{eff} becomes a random variable for each load cycle. The statistical description of the fatigue crack growth driving force can be approximated as

$$\Delta K_{\text{eff}} = \left(Z_{\sigma_{\max}} \bar{\sigma}_{\max}\right)\left(1 - \frac{Z_{\sigma_{\text{op}}} \bar{\sigma}_{\text{op, min}}}{Z_{\sigma_{\max}} \bar{\sigma}_{\max}}\right)[Z_Y \bar{Y}(a)]\sqrt{\pi a} \tag{25}$$

where overbars represent mean values and $Z_{\sigma_{\max}}$, $Z_{\sigma_{\text{op}}}$, and Z_Y are assumed to be log-normal random variables representing the randomness in fatigue load, crack opening stress, and geometry. The variable includes the effects of manufacture tolerance, surface condition, aspect ratio of the crack, etc. Because the crack closure is affected by the load, Eq. (25) may be simplified so that

$$\begin{aligned} \Delta K_{\text{eff}} &= \exp(z_L + z_Y)\bar{\sigma}_{\max}(1 - \bar{\sigma}_{\text{op}}/\bar{\sigma}_{\max})\bar{\psi}(a)\sqrt{\pi a} \\ &= \exp(z_L + z_Y)\overline{\Delta K}_{\text{eff}} \end{aligned} \tag{26}$$

In this simplification, the variation in the maximum load is assumed to be similar to the variation in the crack opening stress, and z_L and z_Y are symmetrical random variables representing the variation due to load and geometry. They can be reasonably assumed to be independent. A single random variable can be used to represent the combined effect of fatigue loading and geometry,

$$Z_K = \exp(z_L + z_Y) = \exp z_K, \quad \text{for} \quad z_K = z_L + z_Y \tag{27}$$

In general, the crack growth may cross several stationary regimes in the crack growth rate with different material parameters C_i and b_i under the variable amplitude fatigue loading condition. The effect of stress on the crack growth may be fully correlated under spectrum loading because a highly loaded sample may have greater chance of being kept in the same condition. It is at least conservative to assume a fully correlated effect in the stress for the crack propagation analysis crossing several material regimes.

Yang and Manning[21] proposed a solution for fully correlated crack growth crossing several material regimes. Their method can be modified to account for the effect of the stress on the fatigue crack growth analysis. In accordance with their solution, the fatigue crack growth can be analyzed based on the mean fatigue loading sequence at the mean stress level. The number of load cycles within each material regime is recorded separately. For each material region, the random variable can be solved as

$$n_i = \bar{n}_i/(Z_K)^{b_i} \tag{28}$$

The total number of fatigue cycles can be solved by

$$n(a) = \sum_{i=1}^{m} \frac{\bar{n}_i}{(Z_K)^{b_i}} + n_0 \tag{29}$$

where n_0 is the number of load cycles below the crack opening stress that do not contribute to the fatigue crack growth (see Fig. 2) and can be treated separately as a random variable.

The random variable Z_K is assumed to follow a log-normal distribution so that

$$F_{Z_K}(x) = \Phi[\ln(x/Z_0)/\sigma_K] = \Phi\left[\ln(x/Z_0)^{1/\sigma_K}\right] = \Phi[\ln y] \tag{30}$$

where Z_0 is the value for 50% probability for the random variable of Z_K. A substitution can be made according to the preceding distribution for

$$Z_K = Z_0 y^{\sigma_K} \tag{31}$$

so that a complete correlated solution can be achieved for $n(a)$ as

$$n(a) = \sum_{i=1}^{m} \frac{\bar{n}_i}{\left(Z_0 y^{\sigma_K}\right)^{b_i}} + n_0 \tag{32}$$

The distribution function of fatigue cycles can be solved for the effect of stress and load by

$$F_K(n) = \int_0^n f_{\text{log normal}}[y(n)]\,\mathrm{d}[y(n)] \tag{33}$$

where $f_{\text{log normal}}$ is the density function for a log-normal distribution. The density function can be solved as

$$f_K(n) = f_{\text{log normal}}[y(n)]\left|\frac{\partial[y(n)]}{\partial n}\right| \tag{34}$$

The central moments up to the second order can be solved based on the crack closure model and the damage accumulation solution.

The distribution density for a given crack size can be approximated using the maximum extreme distribution such as

$$f_{\text{stochastic}}(n) = \alpha \exp\left[-\alpha(n-u) - e^{-\alpha(n-u)}\right] \tag{35}$$

Parameters in this distribution function can be solved according to the central moments

$$u \approx E\{n\} - (0.577/\alpha) \tag{36}$$

$$\alpha = \pi/\sqrt{6\,\text{var}\{n\}} \tag{37}$$

The distribution function is then determined by

$$F_{\text{stochastic}}(n) = \exp\{-\exp[-\alpha(n-u)]\} \tag{38}$$

whereas $f_{\text{stochastic}}$ and f_K can be considered to be independent. A joint probability may be determined by

$$f_{\text{joint}}(n_1, n_2) = f_K(n_1) f_{\text{stochastic}}(n_2) \tag{39}$$

The joint distribution function is determined by

$$F_{\text{joint}}(n) = \int_0^n f_K(n_1)\,\text{d}n_1 \int_0^n f_{\text{stochastic}}(n_2)\,\text{d}n_2 \tag{40}$$

This function is used to evaluate the nonstochastic crack growth when the effects of material inhomogeneity, the fatigue load, and the production quality are considered.

Effect of Initial Flaw and Crack Growth for a Given Time

The distribution of fatigue cycles for a given crack size should be transformed into a flaw size distribution for a given fatigue time. Statistically, when the distribution of the flaw size for a given time is considered, the event $\{a(t) \geq A\}$ is the same as the event of time to crack initiation (TTCI)(t/a). The flaw size distribution (FSD) can then be solved by

$$\text{FSD}(a, a_0 \mid t) = \int_{-\infty}^{t} f_{\text{TCCI}}\,\text{d}t = \text{TTCI}(t \mid a, a_0), \quad \text{for} \quad t = \text{const} \tag{41}$$

This distribution is conditional to a given initial crack size. Effects of load, stress, and stochastic crack propagation are solved in TTCI for a given initial flaw size. The FSD can then be computed by considering effect of the initial crack size distribution in an integration of

$$\text{FSD}(a \mid t) = \int_0^{\infty} \text{TTCI}(t \mid a, a_0) f_{a_0}(a_0)\,\text{d}a_0, \quad \text{for} \quad t = \text{const} \tag{42}$$

where $f_{a_0}(a_0)$ is the distribution of initial flaws. Because the fatigue crack growth is nonlinear, the time distribution depends on crack propagation. When the effect of initial crack distribution has been accounted for, a numerical solution can be used to compute TTCI according to the reversed relation as given in Eq. (41):

$$\text{TTCI}(t \mid a_f) = 1 - \text{FSD}(a_f \mid t), \quad \text{for} \quad a_f = \text{const} \tag{43}$$

When the effect of initial flaw is considered, more than two closure analyses may be made: one for the mean initial crack and the other for the worst 3-σ initial crack. These two computations can be used to account for the effect of initial flaws. The distribution of initial flaws can be assumed, for example, as a log-normal distribution:

$$f_{a_0}(a_0) = \frac{1}{\sqrt{2\pi}\,\sigma_0 a_0} \exp\left\{-\frac{\ln^2(a_0/l_0)}{2\sigma_0^2}\right\} \tag{44}$$

in which

$$l_0 = \frac{E\{a_0\}}{\sqrt{1 + v_{a_0}^2}} \tag{45}$$

where l_0 is the crack size at 50% probability, and

$$\sigma_0 = \sqrt{\ln\left(1 + v_{a_0}^2\right)} \tag{46}$$

which is the log deviation for the initial flaws. In Eq. (46), v_{a_0} is the COV for the initial flaw size defined by

$$v_{a_0} = \frac{\sqrt{\text{var}\{a_0\}}}{E\{a_0\}} \tag{47}$$

The distribution function of initial FSD (IFSD) can be determined by

$$\text{IFSD}(a_0) = \Phi\{\ln(a_0/l_0)/\sigma_0\} \tag{48}$$

This distribution can be used in the evaluation of fatigue crack propagation.

Probability of Failure

When the probabilistic crack growth analysis is realized, the probability of failure (POF) can be determined. The POF is determined by different failure criteria. Crack size, critical stress intensity factor, plastic collapse, etc., can be used as the failure criteria. The simplest one is to use a given crack size. In this case the POF is simply solved by

$$\text{POF}\{t\} = \int_{-\infty}^{t} f_{\text{TCCI}}\,\text{d}t = \text{TTCI}(t \mid a_f) \tag{49}$$

where a_f is the failure criterion.

When the critical stress intensity factor is used as a failure criterion, a state function of failure can be established as

$$g_f = K_{\text{cr}} - K_{\text{max}} \tag{50}$$

where K_{cr} is a critical stress intensity factor, which is a random variable. The POF is determined by

$$\text{POF}\{t\} = \int_{g_f < 0} f_{g_f}(g_f)\,\text{d}g_f \tag{51}$$

The maximum stress intensity factor for a given time is a random variable

$$K_{\text{max}} = S_{\text{max}} Y(a_f) \sqrt{\pi a_f} \tag{52}$$

where S_{max} and Y are random variables for a given crack size a_f. Assume that S_{max} and Y are independent, there are

$$E\{K_{\text{max}}\} = E\left\{S_{\text{max}} Y(a_f) \sqrt{\pi a_f}\right\} = \bar{K}_{\text{max}} \tag{53}$$

$$\text{var}\{K_{\text{max}}\} = \pi a\left(E^2\{S_{\text{max}}\}\text{var}\{Y\} + E^2\{Y\}\text{var}\{S_{\text{max}}\} + \text{var}\{S_{\text{max}}\}\text{var}\{Y\}\right) \tag{54}$$

which gives

$$v_{K_{\text{max}}}^2 = v_{S_{\text{max}}}^2 + v_Y^2 + v_{S_{\text{max}}}^2 v_Y^2 \tag{55}$$

Therefore, the mean value becomes

$$\mu_{g_f} = E\{g_f\} = \bar{K}_{\text{cr}} - \bar{K}_{\text{max}} \tag{56}$$

and the variation is solved by

$$\sigma_{g_f}^2 = \text{var}\{K_{\text{cr}}\} + \text{var}\{K_{\text{max}}\} = v_{K_{\text{cr}}}^2 \bar{K}_{\text{cr}}^2 + v_{K_{\text{max}}}^2 \bar{K}_{\text{max}}^2 \tag{57}$$

Assume that the failure state function follows a normal distribution, the POF can be evaluated by

$$\text{POF}\{t\} = \int_0^{\infty} f_{a(t)} \Phi\left\{-\frac{\mu_{g_f}}{\sigma_{g_f}}\right\} \text{d}a \tag{58}$$

The reliability can be computed using the result of POF,

$$R\{t\} = 1 - \text{POF}\{t\} \tag{59}$$

This function serves as a reference for the reliability management.

Probability of Detecting Crack (PODC)

Inspections can be used as a preventive means against the fatigue damage in an operational condition. Nondestructive inspection (NDI) methods are often used to detect fatigue cracks in a maintenance program. It is useful to analyze the effect of inspection based on the inspection accuracy.

Assume a probability of detection (POD) that represents the capability of a method used to detect cracks. For a given method, a POD curve is usually used to represent the likelihood of detecting a crack as a function of the crack size. Various types of POD curves can be used depending on the type and accuracy of the individual inspection method. As an example, the POD curve proposed by Swift and Connolly[22] is used. This POD curve is assumed to follow a Rayleigh distribution with sensitivity determined by the probability of detecting a specified crack size of $a_{90/95}$. This value is a threshold for which the population of cracks to be detected by using the NDI technique is at a probability of 90% at a lower bound of 95% confidence level. The POD curve is expressed as

$$\mathrm{POD}\{A \le a\} = 1 - \exp\left\{-[a/(0.466a_{90/95})]^2/2\right\} \tag{60}$$

The PODC can be derived according to the POD curve and the probabilistic solution of FSD for a given time:

$$\mathrm{PODC}\{t \le T\} = \int_0^\infty f_{a(T)}(a)\left[\int_{-\infty}^0 f_D(a+D)\,\mathrm{d}D\right]\mathrm{d}a$$

$$= \int_0^\infty f_{a(T)}(a)\mathrm{POD}\{A \le a\}\,\mathrm{d}a \tag{61}$$

This solution describes the capability of an NDI method at a given service time as a function of sensitivity of the NDI technique.

Damage Assessment

An estimation of the extent of damage provides a reference about the status of a group of structural details. A simple method has been proposed in the U.S. Air Force durability method[23] using a binomial distribution so that the number of mean damaged details in a group can be evaluated according to

$$\bar{N}_D(t) = k_0 \times \mathrm{POCI} \times \mathrm{FSD}(a \mid t) \tag{62}$$

where k_0 is the total number of details and POCI is a modification to the original U.S. Air Force solution. The standard deviation for $N_D(t)$ is determined by

$$\sigma_{N_D}^2 = k_0 \times \mathrm{POCI} \times \mathrm{FSD}(a \mid t) \times [1 - \mathrm{FSD}(a \mid t)] \tag{63}$$

In the application, the binomial distribution may be approximated by a log-normal distribution, and the number of structural details with cracks less than a certain size can be estimated for a given service time.

Another useful method is to consider the leading crack in a group of structural details. For a group of similar details, the probability of worst case increases with the increase in size of the group. In this area, order statistics can be used.

Consider random variables $X_1, X_2, \ldots, X_k$ to be independent and identically distributed with the same distribution $F_X(x)$; the variable

$$U_k = \min(X_1, X_2, \ldots, X_k) \tag{64}$$

is of interest when the TCCI is considered for a given crack size. By use of order statistics, the probability of U_k can be computed by

$$F_{U_k}(z) = 1 - [1 - F_X(z)]^k \tag{65}$$

and its density is determined by

$$f_{U_k}(z) = k[1 - F_X(z)]^{k-1} f_X(z) \tag{66}$$

In Eqs. (64–66), k is determined by the number of structural details as well as the POCI. For crack propagation starting from a physical initial flaw, not every initial flaw will lead to a subsequent crack propagation due to the fatigue crack growth threshold. Only details should be considered when the crack growth is possible. Suppose the identical details are originally k_0, the actual details should only be considered for

$$k = k_0 \times \mathrm{POCI}, \quad \text{for} \quad k \ge 1 \tag{67}$$

which will be smaller than k_0.

Probability of Restoration (POR) and POF

When periodic inspections are used to evaluate the status of fatigue damage,[24] repair or replacement may be conducted to restore reliability of the structure once fatigue damage is detected. Extensive investigations ranging from basic probabilistic solutions[25] to more sophisticated Bayesian solutions[26] have been conducted on the effects of inspections and restorations on the reliability of structures. These solutions are often cumbersome for applications. The final results are often doubtful because many solutions give a fully restored structural reliability after each inspection despite that missing a crack in an inspection may significantly compromise the reliability of the structure because fatigue damage is often very detrimental at the time of inspection.

To avoid complications in determining the inspection intervals, a total probability solution is used. This solution is based on an analysis of POF and PODC as a function of the service time. In this solution, all of the sequences are considered, whether or not a crack is detected. It is assumed that once a crack is detected, immediate repair or replacement will be conducted. The restoration is so effective that the fatigue quality can be recovered to the orginal standard.

A probability of restoration (POR) is considered first. Here, POR is the PODC for a given inspection event. At the start of the service, POR is assumed to be

$$\mathrm{POR}_0 = 1 \tag{68}$$

where the subscript denotes the event of inspection. This probability defines a new product state (100% restoration) at time zero. Consequently, POR for the first inspection can be computed according to the schematic shown in Fig. 3,

$$\mathrm{POR}_1 = \mathrm{POR}_0\mathrm{PODC}_{0,1} \tag{69}$$

where $\mathrm{PODC}_{0,1}$ is for the crack started at time zero.

For the second inspection, two cases should be considered. One is for the probability of detecting the crack that is missed in the first inspection. The other is for the probability of detecting the crack that is started after the first inspection, when a crack has been detected and restoration has been performed. The solution for POR at the second inspection becomes

$$\mathrm{POR}_2 = \mathrm{POR}_0\mathrm{PODC}_{0,2}(1 - \mathrm{PODC}_{0,1}) + \mathrm{POR}_1\mathrm{PODC}_{1,2} \tag{70}$$

where $\mathrm{PODC}_{0,2}$ is for the crack started at time zero. $\mathrm{PODC}_{1,2}$ is for the crack started after the first restoration. Additional cases should be considered for the third inspection:

$$\mathrm{POR}_3 = \mathrm{POR}_0\mathrm{PODC}_{0,3}(1 - \mathrm{PODC}_{0,1})(1 - \mathrm{PODC}_{0,2})$$

$$+ \mathrm{POR}_1\mathrm{PODC}_{1,3}(1 - \mathrm{PODC}_{1,2}) + \mathrm{POR}_2\mathrm{PODC}_{2,3} \tag{71}$$

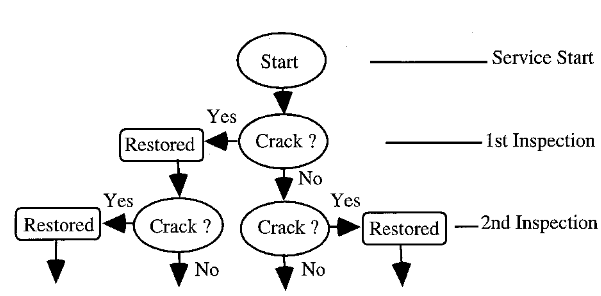


Fig. 3 Schematic of the total probability solution.

Every inspection will lead to a certain probability of repair/replacement. Accordingly, POR can be solved for the kth inspection as

$$\mathrm{POR}_k = \sum_{l=0}^{k-1} \mathrm{POR}_l \mathrm{PODC}_{l,k} \prod_{i=0}^{k-1}(1 - \mathrm{PODC}_{l,i}), \quad \text{for} \quad \mathrm{POR}_0 = 1 \tag{72}$$

In this function, POR_k is the probability of repair/replacement at the kth inspection, which is determined by all of the earlier inspections and restorations. In this solution, $\mathrm{PODC}_{l,k}$ is the POD at the kth inspection for the crack initiated after the lth inspection. $\mathrm{PODC}_{l,k}$ has a value of

$$\mathrm{PODC}_{l,i} = \begin{cases} 0, & \text{for} \quad l = i \\ 1, & \text{for} \quad l > i \end{cases} \tag{73}$$

When the effect of repair/replacement is considered, the evaluation of the POF will be different. The POF before the first inspection can be solved as

$$\mathrm{POF}_0(t) = \mathrm{POR}_0 \mathrm{POF}_{0,t} \tag{74}$$

Immediately after the first inspection, the POF will consist of two events: one is for the probability that a crack has been detected and the restoration has been made; the other is for the probability that the crack has been missed. Solution for the POF after the first inspection becomes

$$\mathrm{POF}_1(t) = \mathrm{POR}_0 \mathrm{POF}_{0,t}(1 - \mathrm{PODC}_{0,1}) + \mathrm{POR}_1 \mathrm{POF}_{1,t} \tag{75}$$

where $\mathrm{POF}_{0,t}$ is for the original crack and $\mathrm{POF}_{1,t}$ is for the crack started after the restoration. When the second inspection is conducted, the solution for the POF becomes

$$\mathrm{POF}_2(t) = \mathrm{POR}_0 \mathrm{POF}_{0,t}(1 - \mathrm{PODC}_{0,1})(1 - \mathrm{PODC}_{0,2}) + \mathrm{POR}_1 \mathrm{POF}_{1,t}(1 - \mathrm{PODC}_{1,2}) + \mathrm{POR}_2 \mathrm{POF}_{2,t} \tag{76}$$

The same as for the solution of POR, POF after the kth inspection can be solved as

$$\mathrm{POF}_k(t) = \sum_{l=0}^{k} \mathrm{POR}_l \mathrm{POF}_{l,t} \prod_{i=l}^{k}(1 - \mathrm{PODC}_{l,i}) \tag{77}$$

where $\mathrm{POF}_{l,t}$ is for the crack started after the lth inspection and restoration and $\mathrm{PODC}_{l,i}$ is for the PODC at the ith inspection for the crack started after the lth inspection. In this solution, every inspection is assumed to be independent. The previous inspections do not affect the inspection capability even for the same detail.

Example and Discussions

To demonstrate the advantages of the probabilistic analysis, a detailed example of a forward fin attachment for an aircraft as shown in Fig. 4 is discussed. The attachment is made of aluminum alloy 3633, with mechanical properties close to 7075. The yield stress is 440 MPa, and the ultimate stress is about 500 MPa. The average Young's modulus and Poisson's ratio for aluminum alloys are used in the crack growth analyses. For this component, a fatigue test was conducted in the 1980s, and the fatigue damage pattern was identified.[27] This example is very interesting because the fatigue life has been demonstrated to be slightly more than twice the design life targets. This component may pass scrutiny with a false reliability picture. To highlight the problem, emphasis will be put on the statistical parameters and their impact on the reliability of the structure. Different scenarios are analyzed to reveal possible problems when the probabilistic fatigue life is considered.

The fatigue test was conducted using the mean spectrum shown in Fig. 5a as solid curves, and the crack propagation is obtained shown in Fig. 5b as symbols. Only one test has been performed because of cost. The test results have been published previously in Ref. 27. The test result showed a small aspect ratio for the initial flaw. This is partly due to the irregular geometry and partly due to the idealization of the initial flaw as a partially elliptical flaw. The actual crack initiation occurs at multiple sites. When a single crack is used to approximate multiple initial cracks, the aspect ratio will be reduced.

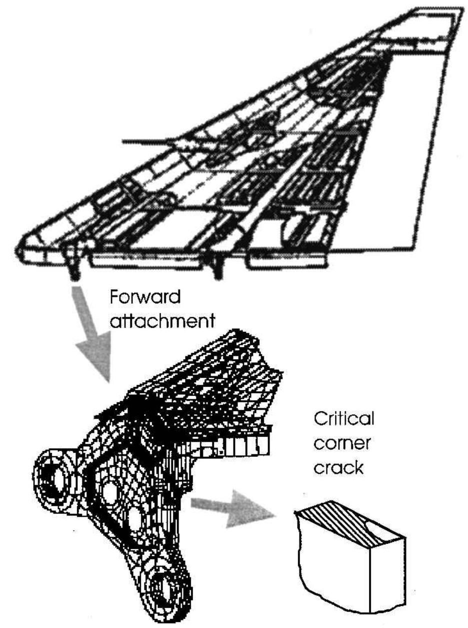


Fig. 4 Finite element model for the forward fin attachment and weight function approximation for the critical corner crack.

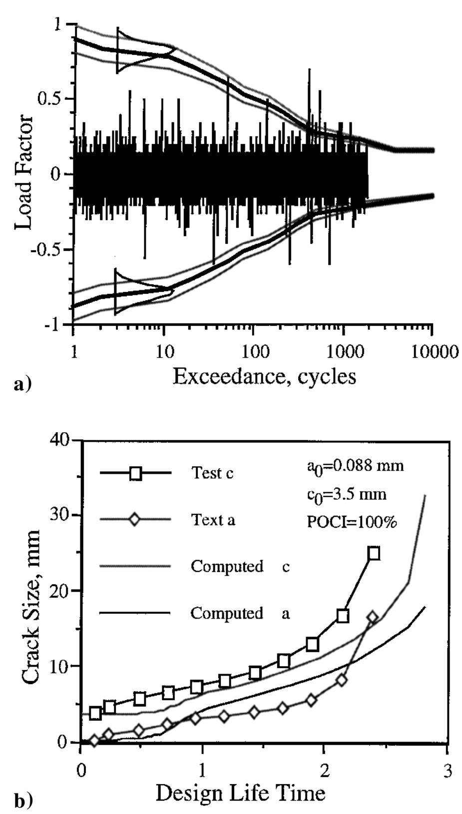


Fig. 5 Load spectrum and the comparison of test and analytical fatigue crack propagation.

A finite element model has been created to analyze stresses in the component as shown in Fig. 4. The boundary conditions for this model are determined according to a global finite element model for both the fin and the fuselage. The finite element analysis provides detailed stress at the stress concentration. The analytical stress concentration agrees well with the experimental crack initiation location. The finite element stress has been used to compute stress intensity factors using weight functions[28] for a corner crack as shown in the insert of Fig. 4. Error in the stress intensity factor solution is estimated to be less then 10%. Fatigue crack growth has been analyzed according to the strip yield model.[29]

The crack closure analysis of crack propagation agrees reasonably well with the test data (curves in Fig. 5b) when the initial flaw is approximated by extrapolating the test data backward to time zero. Both the analytical results and test results show that the fatigue life is more than twice the design life, resulting in a safety factor of two. For the given load and geometry, the deterministic crack closure analysis shows a minimum effective stress intensity of 6.83 MPa $\sqrt{m}$ that is much higher than the average intrinsic crack growth threshold for aluminum alloys (around 1 MPa $\sqrt{m}$), which means a high probability of crack initiation. According to the crack initiation solution, the POCI is predicted to be 100%.

To determine whether or not the obvious safety factor of two can guarantee the reliability of the component, the probabilistic analysis is performed using the procedures given in the preceding sections. The probabilistic crack growth analysis includes four major sources of uncertainties: 1) the initial flaw distribution, 2) the material inhomogeneity, 3) the production uncertainty, and 4) the load uncertainty. The load uncertainty is defined as the uncertainty in the load magnitude, instead of the load sequence. Such an assumption is thought to be relevant for usage because some of the aircraft may be consistently loaded or maneuvered more severely than the others.

The mean initial flaw is assumed to be 3.50 mm on the surface and 0.088 mm deep (Fig. 5b). This initial flaw is similar to a machine scratch. In accord with the inspection of physical initial flaws,[30] the COV may be assumed to be 0.50 for natural initial flaws. The mean intrinsic threshold of aluminum alloys has a value of about 1.00 MPa $\sqrt{m}$ and a COV of 0.18 (Ref. 31). The load spectrum is assumed to be random around the mean value (Fig. 5a) with a COV of 0.10. The uncertainty due to fabrication is characterized with a COV of 0.10. Parameters in the fatigue crack growth rate are determined based on both the small crack growth data[32] and the long crack growth data.[33] The first-order distribution of the crack growth rate is approximated with a smooth change of COV in the fatigue crack growth rate from 0.16 in the Paris regime to 1.50 in the threshold regime.[3] A fully correlated crack growth analysis can be performed for the nonstochastic fatigue crack propagation using these values.

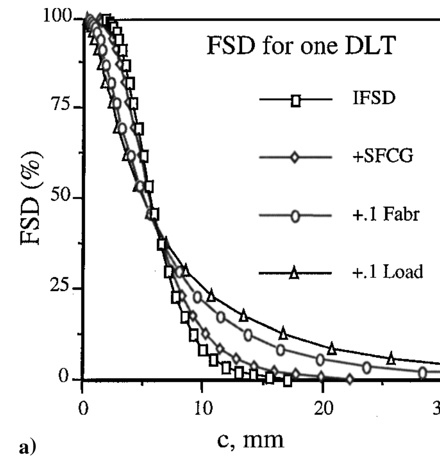
The predicted crack distributions are shown in Fig. 6a at one design life target (DLT). Four results are presented. Results denoted IFSD are the analytical FSD values when only the effect of initial flaw distribution is considered. In this case, the crack growth is assumed to be deterministic. Results denoted +SFCG mean that the effect of material inhomogeneity has been added. Results denoted +0.1 Fabr indicate the uncertainty in fabrication is added. The COV for fabrication is an assumed value for the purpose of parameter analyses. Accurate value may be measured from stress variations in the assembled components. The last result in Fig. 6 is denoted as +0.1 Load. This is the result when the uncertainty in load severity is added. Again, an accurate value could be measured during use of the aircraft.

It is shown that the fatigue crack propagation is affected by uncertainties in both initial flaw sizes and material inhomogeneity. The consistency in production and the change in load severity also affect considerably the probabilistic fatigue crack growth process. These variables especially affect the low tail of distribution. The low tail of distribution is of great concern in the reliability management of structures. For the example as shown in Fig. 6b, material inhomogeneity, production inconsistency, and change in load severity will significantly change the failure probability at the low end. These results indicate that the original DLT may not be satisfied with reasonable reliability.

A rough estimation of the extent of fatigue damage can be made for a fleet of aircraft having the same component. In accord with the probabilistic parameters and the probabilistic crack growth results as shown in Fig. 6, the mean extent of damage is estimated for the half DLT and one DLT for a fleet size of 100. The predicted results are shown in Fig. 7. This analysis shows that the fatigue damage is severe at the forward fin attachment. For example, there will be an average 30 out of 100 aircraft containing surface cracks at the forward fin attachment with a size as large as 5 mm at the half of DLT. For one DLT, the number will be increased to 50.

For a more detailed analysis, a solution for the POF is required. The critical stress intensity factor for a partially through crack is used as the failure criterion. The estimated mean critical stress intensity factor is 32 MPa $\sqrt{m}$, and the scatter is characterized by a COV of 0.14 (Ref. 34). The computed POFs according to this criterion are given in Fig. 8a. Order statistics has been applied to compute POFs for the worst case for different fleet sizes. The worst POF increases with the increase in the fleet size. This is in agreement with common sense in that the more flawed components are used, the more likely the worst case will occur. Such information is very useful for the management of the whole fleet in that the leading damage may be estimated for similar service conditions.

Figure 8a shows that the worst fatigue damaged aircraft cannot survive one DLT for a fleet size larger than 100. When there are no other options, inspections may be arranged to detect the crack and to make restoration to prevent the failure. Figure 8b shows predicted results for the probability of detecting leading cracks as a function of service time for different fleet sizes. The inspection is assumed to have a capability of $a_{90/95} = 20$ mm in the POD function as given in Eq. (60). This inspection accuracy is typical for many of the on-field, nondestructive eddy current inspection methods. The PODC



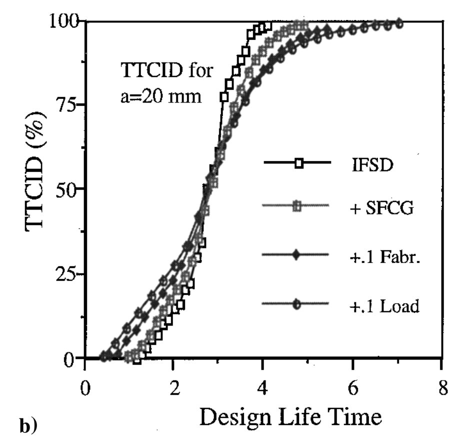


Fig. 6 FSD and TTCID as functions of random variables when different factors are involved.

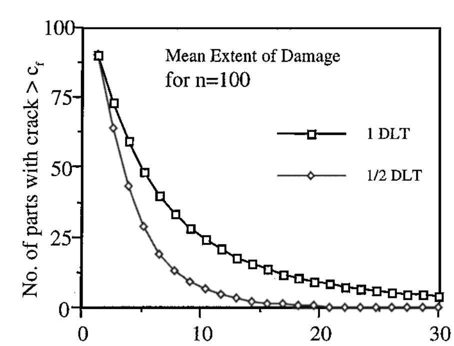


Fig. 7 Examples of the extent of damage.

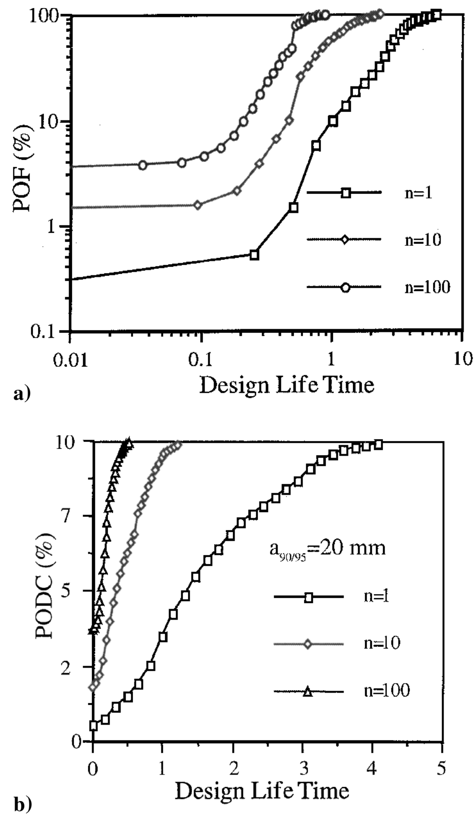


Fig. 8 POF and PODCs for different fleet sizes.

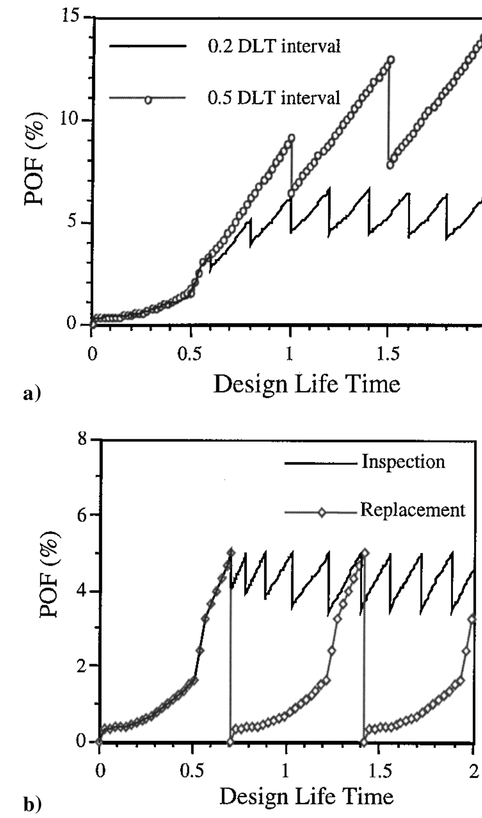


Fig. 9 Comparison of constant interval and variable interval inspections.

results as shown in Fig. 8b indicate that there is a high probability that the leading crack may be detected in the inspection.

Suppose an inspection procedure is planned according to the deterministic method. A conservative inspection interval may be determined at one half of DLT because the fatigue test has demonstrated a fatigue life of about twice the DLTs. It seems that this inspection program can secure at least one DLT. However, the probability analysis gives a different picture because the result in Fig. 8a shows that there is a very high probability that the leading damaged component may fail before one DLT, especially when the size of fleet is large.

According to the preceding sections, probabilistic predictions of POF can be made for the half DLT interval inspection strategy. The predictions are shown in Fig. 9a as symbols. The probabilistic analyses show a very high POF, close to 9% for one DLT according to the half DLT interval inspection method. POF is very high even after the inspection and restoration. The first inspection seems to have almost no effect on POF. The high POF after the inspections seems to be due to the significant effect of the missed cracks in the inspection.

The increase of inspection frequency is a convenient way to increase reliability of the aircraft. It has been used widely in service when fatigue problems have become severe. By use of the present procedure, how changes in the inspection interval may affect structural reliability can be analyzed. An example is shown in Fig. 9a (solid curve) for a reduced 0.2 DLT interval inspection. The reduced inspection interval improves the reliability considerably. The POF is reduced to less than 6.5% up to two DLTs. Notice that inspections still have a negligible effect on the POF for the life duration less than 0.6 DLT even when the inspection frequency is increased.

A variable interval inspection procedure may be designed according to a given reliability requirement. For example, for a maximum POF of 5%, the inspection intervals can be solved according to the total probability solution as given in Eqs. (68–77). The predicted result is shown in Fig. 9b (solid curve). A very interesting outcome of the variable interval solution is that no inspection is required before 0.7 DLT. However, the second inspection should be very close to the first one because missing a crack in the first inspection may lead to considerable reduction in reliability of the structure. The inspection interval is gradually increased after restoration. The required inspection interval seems to be periodic in a class of 0.2 DLT.

This probabilistic prediction suggests another option to make all restorations at a constant interval of 0.7 DLT without involving inspections. Suppose restorations are conducted after every 0.7 DLT to recover the component to its original quality; the reliability function is predicted and shown in Fig. 9b (curve with symbols). The prediction shows the advantage that reliability can be significantly improved after the restoration. Another benefit is that no cost is involved because of inspections and the ensuing replacement, repair, and grounding. The inspection option may have the advantage of leaving some components that do not need restoration, but the method has the cost of frequent inspections and grounding and a sustained high POF because critical cracks may be missed in the inspection. It may be argued that the reliability target is still satisfied even though POF may be high after the inspection. A full management cost analysis should be made before any conclusion can be made to compare these options.

To analyze the full cost picture, a comparison should be made for the accumulated PORs for different options. Figure 10a shows a comparison among the cumulative restoration curves for constant interval inspection, variable interval inspection, and constant restoration options. The cumulative probability of restoration is the sum of all the restoration actions required during a given service time. It is closely related to the cost of maintenance. Here, cost of inspections and corresponding grounding is not included. In this scenario, the constant restoration gives, as can be expected, the highest cost at 0.7 DLT. The constant interval inspection cost less, whereas the lowest cost method is the variable interval inspection. The cost of maintenance changes when a longer service time is considered for 1.2 ∼ 1.4 DLT. In this case, the constant restoration becomes the cheapest option, whereas the constant interval inspection becomes most costly and the variable interval inspection costs slightly less than the constant interval inspection.

Differences in the cost function become even bigger when the cost of inspections and grounding are included. For the same reliability requirement, there are six inspections and grounding required for the constant interval inspection option when the service time reaches

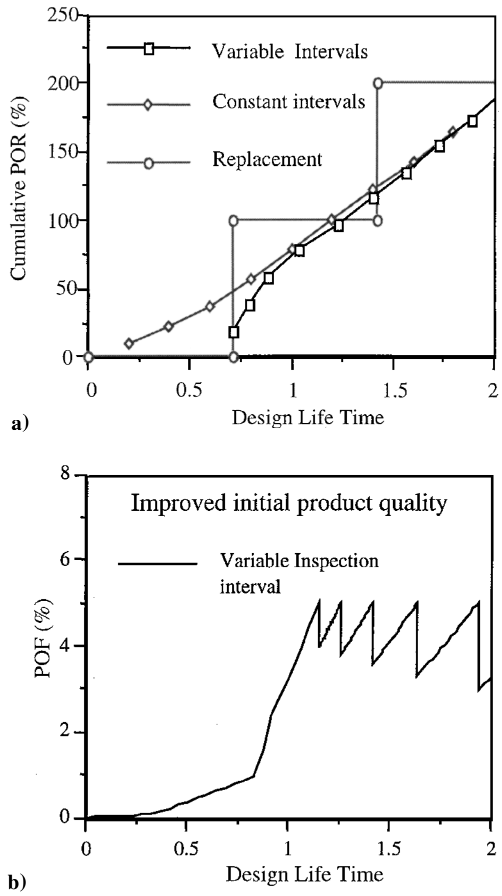


Fig. 10 Comparison of various strategies.

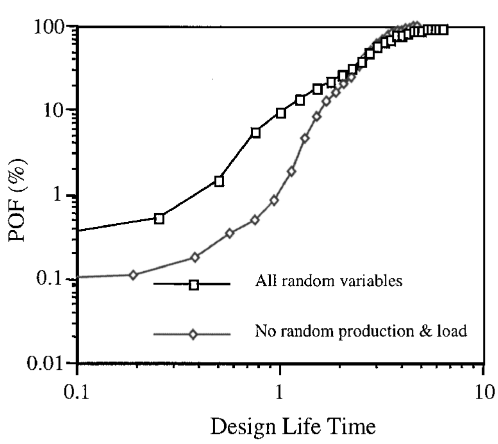


Fig. 11 Comparison of POF whether or not the randomness in production and loading is considered.

1.4 DLT. The number of required inspections is five for the variable interval inspection option. For the constant restoration option, there is only one replacement and grounding during the entire 1.4 DLT service life of the aircraft. By use of this analysis, it is obvious that use of inspections cannot be rationalized as a major option to keep reasonable reliability of the structures.

The probabilistic analyses show that the cost of service and maintenance is not solely determined by design (control of the mean stress). When design parameters are not changed, reliability of the component can be increased if improvements can be made on the initial quality of the component by reducing the initial flaw size and increasing consistency in both production and material. For example, if the initial flaw sizes are reduced to half and improvement in the consistency in production and material is doubled, the POF can be predicted as shown in Fig. 10b for a variable interval inspection option. In this case, the POF will be considerably reduced. This example shows that the initial DLT may be achieved with a given POF of 5% without implementation of inspections and restorations if improvements can be made in initial fatigue quality of the component. It is, however, still costly to operate the component beyond the DLT because the required inspection intervals are very small and grounding and restorations will be frequent.

Finally, note that a strict distinction must be made between the probabilistic fatigue crack growth under service conditions and under laboratory conditions. In the laboratory, fatigue loading can be accurately controlled, and stress at the crack location can be monitored and evaluated. In such a condition, the effect of uncertainties in production and service loading is excluded. Therefore, the POF will be different. Figure 11 shows a comparison between the predicted POF in service conditions and the predicted POF in laboratory conditions. At a fatigue life of 0.7 DLT, the POF is about 5.0% under service conditions whereas it is only 0.4% under laboratory conditions because the randomness in both production and loading is excluded in laboratory conditions. The prediction agrees with the empirical practice to have a low POF in a range of 10^{-3} for the high maneuver airplanes when the prediction is based on fatigue tests. Such practices may lead to a high POF under service conditions even though fatigue tests may indicate adequate reliability. Apparently, the high reliability required in the fatigue tests does not guarantee reasonable reliability of structures under service conditions even though materials, initial flaws, geometry, and the mean fatigue loading are the same.

Conclusion

The damage tolerance method used in the industry can prevent many of reliability-related structural problems for certification of new structures. However, it is difficult to judge the economy of usage based on deterministic damage tolerance analyses where in-service management and maintenance are concerned. The deterministic analysis may create misleading information about the quality of a product.

To understand the total economy of a product, probability analyses are essential. Compared to the deterministic methods, probabilistic analyses may provide a different view of the fatigue behavior of a product. Probabilistic predictions are crucial in determining a proper strategy for new as well as aging products. A probabilistic analytical procedure has been introduced for the analysis of fatigue crack growth based on conventional as well as some novel solutions. The model is established around a crack closure model so that a sound physical mechanism is used throughout the analysis to account for many aspects of fatigue crack growth behaviors.

An example has been provided to demonstrate the significance of probabilistic analyses. Different scenarios have been analyzed for this example. It has been shown that the probabilistic model can help to improve the understanding of problems in both fatigue crack growth analysis and structural life management methods so that effective strategies can be determined to deal with fatigue problems in the critical structural details under service conditions.

Acknowledgments

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References

[1]Elber, W., "Fatigue Crack Closure Under Cyclic Tension," *Engineering Fracture Mechanics*, Vol. 2, No. 1, 1970, pp. 37–45.

[2]Wang, G. S., and Blom, A. F., "A Strip Model for Fatigue Crack Growth Predictions Under General Load Conditions," *Engineering Fracture Mechanics*, Vol. 40, No. 3, 1991, pp. 507–533.

[3]Paris, P. C., and Erdogan, F., "A Critical Analysis of Crack Propagation Laws," *Journal of Basic Engineering, Series D*, Vol. 85, 1963, pp. 528–534.

[4]Kozin, F., and Bogdanoff, J. L., "Cumulative Damage Model for Fatigue Crack Growth Based on Reaction Rate Theory—II," *Engineering Fracture Mechanics*, Vol. 41, No. 5, 1992, pp. 873–896.

[5]Ohji, K., Ogura, K., and Ohkubo, Y., *Engineering Fracture Mechanics*, Vol. 7, No. 3, 1975, pp. 457–464.

[6]Wang, G. S., Palmberg, B., and Blom, A. F., "Stress State-Related Fatigue Crack Growth Under Spectrum Loading," *Fatigue and Fracture in Engineering Materials and Structures*, Vol. 15, No. 7, 1992, pp. 695–712.

[7]Wang, G. S., "The Plasticity Aspect of Fatigue Crack Growth," *Engineering Fracture Mechanics*, Vol. 46, No. 6, 1993, pp. 909–930.

[8]Dugdale, K. D., "Yielding of Steel Sheets Containing Slits," *Journal of Mechanics and Physics of Solids*, Vol. 8, No. 2, 1960, pp. 100–104.

[9]Newman, J. C., Jr., "Prediction of Fatigue Crack Growth Under Variable-Amplitude and Spectrum Loading Using a Closure Model," American Society for Testing and Materials STP 761, 1982, pp. 255–277.

[10]Doecker, H., and Bachmann, V., "Determination of Crack Opening Load by Use of Threshold Behavior," American Society for Testing and Materials STP 982, 1988, pp. 247–259.

[11]Carpinteri, A. (ed.), *Handbook of Fatigue Crack Propagation in Metallic Structures*, Elsevier Science B. V., Amsterdam, 1994, pp. 1721–1765.

[12]Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, McGraw–Hill, New York, 1984, pp. 89–122.

[13]Kozin, F., and Bogdanoff, J. L., "Probabilistic Models of Fatigue Crack Growth: Results and Speculations," *Nuclear Engineering and Design*, Vol. 115, 1989, pp. 143–171.

[14]Sobczyk, K., and Spencer, B. F., Jr., *Random Fatigue—From Data to Theory*, Academic, New York, 1992, pp. 42–44.

[15]Bogdanoff, J. L., and Kozin, F., "On Nonstationary Cumulative Damage Models," *Journal of Applied Mechanics*, Vol. 49, No. 1, 1982, pp. 37–42.

[16]Miner, M. A., "Cumulative Damage in Fatigue," *Journal of Applied Mechanics*, Vol. 12, No. 3, 1945, pp. 159–164.

[17]Miles, J. W., "On Structural Fatigue Under Random Loading," *Journal of the Aeronautical Sciences*, Vol. 21, 1954, pp. 753–762.

[18]Jacoby, G. H., and Nowack, H., "Comparison of Scatter Under Program and Random Loading and Influencing Factors," American Society for Testing and Materials STP 511, 1972, pp. 61–72.

[19]Yang, J. N., and Manning, S. D., "A Simple Second Order Approximation for Stochastic Crack Growth Analysis," *Engineering Fracture Mechanics*, Vol. 53, No. 5, 1996, pp. 677–686.

[20]Tanaka, H., and Tsurui, A., "Reliability Degradation of Structural Components in the Process of Fatigue Crack Propagation Under Stationary Random Loading," *Engineering Fracture Mechanics*, Vol. 27, No. 5, 1987, pp. 501–516.

[21]Yang, J. N., and Manning, S. D., "Stochastic Crack Growth Analysis Methodologies for Metallic Structures," *Engineering Fracture Mechanics*, Vol. 37, No. 5, 1990, pp. 1105–1124.

[22]Swift, R., and Connolly, M. P., "Some Statistical Aspects of NDT," 20th Annual British Conf. on Non-Destructive Testing, Paper 7A, 1985.

[23]Manning, S. D., and Yang, J. N., "USAF Durability Design Handbook: Guidelines for theAnalysis and Design of Durable Aircraft Structures," U.S. Air Force Research Lab., AFWAL-TR-83-3027, Wright–Patterson AFB, OH, Jan. 1984.

[24]Wittaker, I. C., and Saunders, S. C., "Application of Reliability Analysis to Aircraft Structures Subjected to Fatigue Crack Growth and Periodic Inspections," U.S. Air Force Materials Lab., AFML-TR-73-92, Wright–Patterson AFB, OH, 1973.

[25]Yang, J.-N., and Trapp, W. J., "Reliability Analysis of Aircraft Structures Under Random Loading and Periodic Inspection," *AIAA Journal*, Vol. 12, No. 12, 1974, pp. 1623–1630.

[26]Deodatis, G., Fujimoto, Y., Ito, S., Spencer, J., and Itagaki, H., "Non-Periodic Inspection by Bayesian Method," *Probabilistic Engineering Mechanics*, Vol. 7, No. 4, 1992, pp. 121–215.

[27]Palmberg, B., "Damage Tolerance Assessment of Four Versions for the Main Wing Attachment Frame and Some Parts of the JA-Fin for the Aircraft 37 Viggen," Aeronautical Research Inst. of Sweden, FFA TN 1990-30, Bromma, Sweden, 1990.

[28]Wang, G. S., "A Generalised WF Solution for Mode I 2D Part-Elliptical Cracks," *Engineering Fracture Mechanics*, Vol. 45, No. 2, 1993, pp. 177–208.

[29]Wang, G. S., "Analysing the Fatigue Crack Growth in Structural Details," *Engineering Fracture Mechanics*, Vol. 53, No. 3, 1996, pp. 345–361.

[30]Laz, P. J., and Hillberry, B. M., "Fatigue Life Prediction from Inclusion Initiated Cracks," *International Journal of Fatigue*, Vol. 20, No. 4, 1998, pp. 263–270.

[31]Liaw, P. K., and Logsdon, W. A., "Near-Threshold Fatigue Crack Growth Behavior in Metals," *Acta Metallurgica*, Vol. 31, No. 10, 1983, pp. 1581–1587.

[32]Newman, J. C., Jr., and Edwards, P. R., "Short-Crack Growth Behaviour in an Aluminium Alloy—An AGARD Cooperative Test Programme," R-732, AGARD, Dec. 1988.

[33]Palmberg, B., Olsson, M.-O., Boman, P.-O., and Blom, A. F., "Damage Tolerance Assessment of the Fighter Aircraft 37 Viggen Main Wing Attachment," *Journal of Aircraft*, Vol. 30, No. 3, 1993, pp. 377–381.

[34]Schutz, W., "Treatment of Scatter of Fracture Toughness Data for Design Purposes," *Practical Applications of Fracture Mechanics*, edited by H. Liebowitz, AG-257, AGARD, 1980.

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